Q No		Answer	Marks	Comments
(1)	(a)	$\lvert A-\lambda I\rvert=0$ $\left\lvert\begin{bmatrix}1&2\\5&4\end{bmatrix}-\lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\rvert=0$ $\begin{vmatrix}1-\lambda&2\\5&4-\lambda\end{vmatrix}=0$ $(1-\lambda)(4-\lambda)-10=0$ $\lambda^2-5\lambda-6=0$ $(\lambda-6)(\lambda+1)=0$ $\therefore$ $\lambda=6$ or $\lambda=-1$	30	
		Taking $\lambda_1=-1$ and $\lambda_2=6$ $AX_1=\lambda_1X_1$ $\begin{bmatrix}1&2\\5&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=-\begin{bmatrix}x\\y\end{bmatrix}$ $\begin{bmatrix}x+2y\\5x+4y\end{bmatrix}=\begin{bmatrix}-x\\-y\end{bmatrix}$ $\begin{matrix}x+2y=0\\5x+5y=0\end{matrix}$ $\therefore$ $x+y=0$ Take $x=t$ then $y=-t$ where t is a parameter $\therefore$ $X_1=\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}t\\-t\end{bmatrix}=t\begin{bmatrix}1\\-1\end{bmatrix}$ $AX_2=\lambda_2X_2$ $\begin{bmatrix}1&2\\5&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=6\begin{bmatrix}x\\y\end{bmatrix}$ $\begin{bmatrix}x+2y\\5x+4y\end{bmatrix}=\begin{bmatrix}6x\\6y\end{bmatrix}$ $\begin{matrix}2y=5x\\2y=5x\end{matrix}$ $y=\frac{5}{2}x$ $X_2=\begin{bmatrix}x\\\frac{5x}{2}\end{bmatrix}$ $X_2=\frac{x}{2}\begin{bmatrix}2\\5\end{bmatrix}$ Take $r=\frac{x}{2}$ where t is a parameter $X_2=r\begin{bmatrix}2\\5\end{bmatrix}$	40	
		Suppose $\alpha\begin{bmatrix}1\\-1\end{bmatrix}+\beta\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $\begin{bmatrix}\alpha+2\beta\\-\alpha+5\beta\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $\alpha+2\beta=0$ $-\alpha+5\beta=0$ $\alpha=\beta=0$ $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}2\\5\end{bmatrix}$ are linearly independent	15	

Q No		Answer	Marks	Comments
		$U=\begin{bmatrix}1 & 2\\ -1 & 5\end{bmatrix}$ $D=\begin{bmatrix}-1 & 0\\ 0 & 6\end{bmatrix}$ $U^{-1}=\frac{1}{5+2}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}=\frac{1}{7}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}$ $UDU^{-1}=\begin{bmatrix}1 & 2\\ -1 & 5\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & 6\end{bmatrix}\left(\frac{1}{7}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}\right)$ $U^{-1}=\frac{1}{7}\begin{bmatrix}-1 & 12\\ 1 & 30\end{bmatrix}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}$ $U^{-1}=\frac{1}{7}\begin{bmatrix}-5+12 & 2+12\\ 5+30 & -2+30\end{bmatrix}=\begin{bmatrix}1 & 2\\ 5 & 4\end{bmatrix}=A$ $A=UDU^{-1}$	25	
		$A^n=(UDU^{-1})^n=(UDU^{-1})(UDU^{-1}).......(UDU^{-1})(UDU^{-1})$ $A^n=UD(U^{-1}U)D(U^{-1}U).....(U^{-1}..U)D(U^{-1}U)DU^{-1}$ $A^n=UD(I)D(I).....(I)D(I)DU^{-1}$ $A^n=UD^nU^{-1}$ $A^n=\begin{bmatrix}1 & 2\\ -1 & 5\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & 6\end{bmatrix}^n\left(\frac{1}{7}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}\right)$ $A^n=\begin{bmatrix}1 & 2\\ -1 & 5\end{bmatrix}\begin{bmatrix}(-1)^n & 0\\ 0 & 6^n\end{bmatrix}\left(\frac{1}{7}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}\right)$ $A^n=\begin{bmatrix}(-1)^n & 2.6^n\\ -(-1)^n & 5.6^n\end{bmatrix}\left(\frac{1}{7}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}\right)$ $A^n=\frac{1}{7}\begin{bmatrix}(-1)^n & 2.6^n\\ -(-1)^n & 5.6^n\end{bmatrix}\begin{bmatrix}5 & -2\\ 1 & 1\end{bmatrix}$ $A^n=\frac{1}{7}\begin{bmatrix}5(-1)^n+2.6^n & -2(-1)^n+2.6^n\\ -5(-1)^n+5.6^n & 2(-1)^n+5.6^n\end{bmatrix}$	40	

Q No		Answer	Marks	Comments
	(b)	$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$ Applying $C_1 \to C_1 + C_2 - 2C_3$ $= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$ Taking $a^2+b^2+c^2$ as a common factor from C_1 $= (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ Applying $R_3 \to R_3 - R_2$ $R_2 \to R_2 - R_1$ $= (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 0 & (b^2-a^2) & c(a-b) \\ 0 & (c^2-b^2) & a(b-c) \end{vmatrix}$ Taking common factors $(a-b)$ and $(b-c)$ from C_2 and respectively $= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 1 & a^2 & bc \\ 0 & -(b+a) & c \\ 0 & -(c+b) & a \end{vmatrix}$ Expanding along C_1 $= (a^2+b^2+c^2)(a-b)(b-c)[-a^2-ab+bc+c^2]$ $= (a^2+b^2+c^2)(a-b)(b-c)[(c-a)(c+a)+b(c-a)]$ $= (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$	50	Expanding, without applying column & raw operations no marks

Q No		Answer	Marks	Comments
2	(a)	$x = at^2 \quad , y = 2at \qquad \frac{dy}{dt} = 2a \, , \frac{dx}{dt} = 2at$ $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} \qquad \frac{dy}{dx} = 2a\frac{1}{2at} = \frac{1}{t}$ $\frac{dy}{dx}m_N = -1 \quad \frac{1}{t}m_N = -1 \quad m_N = -t$ The equation of the normal at $(at^2, 2at)$ is $y - 2at = -t(x - at^2)$ $y + tx = 2at + at^3$	20	
		If $P(h,k)$ lies any arbitrary normal then $k + th = 2at + at^3$ $at^3 + (2a - h)t + k = 0 - - - - - - [1]$ Since the above equation has three roots then generally there are three normal passing through the point $P(h,k)$ Suppose the roots of the cubic equation are t_1 , t_2 , t_3 $(t - t_1)(t - t_2)(t - t_3) = 0$ $t^3 - (t_1 + t_2 + t_3)t^2 + (t_1t_2 + t_2t_3 + t_3t_1)t - t_1t_2t_3 = 0..[2]$ Since the equations [1] and [2] are the same the corresponding coefficients are equal $(t_1 + t_2 + t_3) = 0$, $(t_1t_2 + t_2t_3 + t_3t_1) = \frac{2a-h}{a}$, $t_1t_2t_3 = \frac{k}{a}$ The gradient of a normal is $-t$. Since two normal are perpendicular $(-t_1)(-t_2) = -1 \quad t_1t_2 = -1$ $t_3 = -\frac{k}{a} t_3$ satisfy the equation [1] $a\left(\frac{-k}{a}\right)^3 + (2a - h)\left(\frac{-k}{a}\right) - k = 0$ Since $k \neq 0$ $k^2 + a(2a - h) + a^2 = 0$ $k^2 = a(h - 3a)$ Hence the point $P(h,k)$ lies on the curve $y^2 = a(x - 3a)$	70	

Q No		Answer	Marks	Comments
2	(b)	$\frac{x+1}{2}=\frac{\mathrm{y}+3}{2}=\frac{\mathrm{z}-4}{-1}=\mathrm{k}$ $x=2k-1\,, y=2k-3\,, z=-k+4$ Since this point lies on the plane $x+2y-2z-9=0$ $(2k-1)+2(2k-3)-2(4-k)-9=0$ $k=3$ The intersection point is $(2.3-1\,,\ 2.3-3\,,\ 4-3)=$ $(5\,,3\,,1)$	30	
		Suppose $(a,b\,,c)$ be any point then the direction ratios of the line perpendicular to the plane $x+2y-2z-9=0$ are $1\,,2\,,-2$ therefore the equation of the line passing through the point$(a,b\,,c)$ perpendicular to the above plane $\frac{x-a}{1}=\frac{\mathrm{y}-\mathrm{b}}{2}=\frac{\mathrm{z}-\mathrm{c}}{-2}$ Any point on the above line $(a+\mu\,,\ b+2\mu\,,\ c-2\mu)$ Suppose this point lie on the plane $x+2y-2z-9=0$ Then $a+\mu+2(b+2\mu)-2(c-2\mu)-9=0$ $\mu=\frac{-(a+2b-2c-9)}{9}$ Therefore the perpendicular distance from the point $(a,b\,,c)$ to the plane $x+2y-2z-9=0$ is $=\sqrt{(a+\mu-a)^2+(b+2\mu-b)^2+(c-2\mu-c)^2}$ $=\sqrt{\mu^2+(2\mu)^2+(-2\mu)^2}=\sqrt{9\mu^2}=3\lvert\mu\rvert$ $=3\left\lvert\frac{a+2b-2c-9}{9}\right\rvert=\frac{\lvert a+2b-2c-9\rvert}{3}$ $\frac{\lvert(2k-1)+2(2k-3)-2(4-k)-9\rvert}{3}=8$ $\frac{\lvert 8k-24\rvert}{3}=8$ $\frac{\lvert k-3\rvert}{3}=1$ $k=6$ or $k=0$ therefore the points are $(2.6-1\,,2.6-3\,,4-6)=(11\,,9\,,-2)$ and $(-1\,,-3\,,4)$	80	The formula for the perpendicular distance to a plane from a point cannot be used since it is not in the course material. If it is used half of the marks will be deducted.

Q No		Answer	Marks	Comments
3	(a)	ABCD is a tetrahedron, AB = BC = CD = DA = BD = AC Draw a line MS parallel BC BS = SD (construction) AM = $\sqrt{3}a$ [Where 2a = length of the side of the tetrahedron] MS = a [Construction mid points theorem] AS = $\sqrt{3}a$ From the triangle AMS; $AS^2 = AM^2 + SM^2 - 2AM.MS\cos S\hat{M}A$ $(\sqrt{3}a)^2 = (\sqrt{3}a)^2 + a^2 - 2\sqrt{3}aacosSMA$ $cosSMA = \frac{a^2}{2\sqrt{3}a^2} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$ $SMA = cos^{-1}\frac{\sqrt{3}}{6}$ Since BC // SM $\therefore$ The angle between AM and BC is $cos^{-1}\frac{\sqrt{3}}{6}$	35	
	(b)	Let $\boldsymbol{a}$, $\boldsymbol{b}$ two vectors Scalar product If $\boldsymbol{a} \neq \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{0}$ then $\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\boldsymbol{cos}\theta$ is defined as the scalar product of $\boldsymbol{a}$ and $\boldsymbol{b}$, where θ is the angle between $\boldsymbol{a}$, $\boldsymbol{b}$ **.** If $\boldsymbol{a} = \mathbf{0}$ or $\boldsymbol{b} = \mathbf{0}$ then $\mathbf{0}$ is defined as the vector product of $\boldsymbol{a}$, $\boldsymbol{b}$ Vector product If $\boldsymbol{a} \neq \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{0}$ then $\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert sin\theta\boldsymbol{n}$ is defined as the vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$,where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. The direction is that of the unit vector $\boldsymbol{n}$ which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$ such that $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{n}$ form a right handed system. If $\boldsymbol{a} = \mathbf{0}$ or $\boldsymbol{b} = \mathbf{0}$ then $\mathbf{0}$ is defined as the vector product of $\boldsymbol{a}$, $\boldsymbol{b}$.	25	If the vector notations are not used half of the marks will be deducted.

Q No		Answer	Marks	Comments
	(A)	$\boldsymbol{a}\times\boldsymbol{b} = 3\boldsymbol{a}\times\boldsymbol{c} \quad \boldsymbol{a}\times\boldsymbol{b} - 3\boldsymbol{a}\times\boldsymbol{c} = \boldsymbol{0}$ $\boldsymbol{a}\times(\boldsymbol{b}-3\boldsymbol{c}) = \boldsymbol{0}$ $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = 3\boldsymbol{c}$ or $\boldsymbol{a}$ is parallel to $\boldsymbol{b}-3\boldsymbol{c}$ $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = 3\boldsymbol{c}$ or $\boldsymbol{b}-3\boldsymbol{c} = k\boldsymbol{a}$ where is a scalar $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = 3\boldsymbol{c} + 0\boldsymbol{a}$ or $\boldsymbol{b} = 3\boldsymbol{c} + k\boldsymbol{a}$ $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = 3\boldsymbol{c} + k\boldsymbol{a}$ $\lvert\boldsymbol{a}\rvert = 5\,,\ \lvert\boldsymbol{b}\rvert = 3\,,\ \lvert\boldsymbol{c}\rvert = \sqrt{3}\ \ \boldsymbol{b}\cdot\boldsymbol{c} = 0$ and $\boldsymbol{b}-3\boldsymbol{c} = k\boldsymbol{a}$ $(\boldsymbol{b}-3\boldsymbol{c})\cdot(\boldsymbol{b}-3\boldsymbol{c}) = k\boldsymbol{a}\cdot k\boldsymbol{a}$ $\lvert\boldsymbol{b}\rvert^{2} - 6\boldsymbol{c}\cdot\boldsymbol{b} + 9\lvert\boldsymbol{c}\rvert^{2} = k^{2}\lvert\boldsymbol{a}\rvert^{2}$ $k^2 = \frac{36}{25} \quad k = \pm\frac{6}{5}$ $\boldsymbol{b}-3\boldsymbol{c} = k\boldsymbol{a}$ $\boldsymbol{c}\cdot\boldsymbol{b} - 3\boldsymbol{c}\cdot\boldsymbol{c} = k\boldsymbol{a}\cdot\boldsymbol{c}$ $\boldsymbol{c}\cdot\boldsymbol{b} - 3\boldsymbol{c}\cdot\boldsymbol{c} = k\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{c}\rvert cos\theta$ Where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{c}$ **.** $cos\theta = \frac{-9}{5k\sqrt{3}}$ When $k = \frac{6}{5} \quad \theta = \frac{5\pi}{6}$ When $k = -\frac{6}{5} \quad \theta = \frac{\pi}{6}$ $\boldsymbol{b}-3\boldsymbol{c} = k\boldsymbol{a}$ $\boldsymbol{b}\cdot\boldsymbol{b} - 3\boldsymbol{c}\cdot\boldsymbol{b} = k\boldsymbol{a}\cdot\boldsymbol{b}$ $\boldsymbol{b}\cdot\boldsymbol{b} - 3\boldsymbol{c}\cdot\boldsymbol{b} = k\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert cos\beta$ Where is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ **.** $cos\beta = \frac{3}{5k}$ When $k = \frac{6}{5} \quad \beta = \frac{\pi}{3}$ When $k = -\frac{6}{5} \quad \theta = \frac{2\pi}{3}$	90	
	(B)	$\overrightarrow{OA} = 2\mu\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\ ,\overrightarrow{OB} = \mu\mathbf{i} + (\mu-1)\mathbf{j} + \mathbf{k}$ $\overrightarrow{OC} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{OA}\times\overrightarrow{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2\mu & 3 & 2 \\ \mu & \mu-1 & 1 \end{vmatrix}$	45	

Q No		Answer	Marks	Comments
		$= (3-2\mu+2)\mathbf{i} - (2\mu-2\mu)\mathbf{j} + (2\mu^2-2\mu-3\mu)\mathbf{k}$ $= (5-2\mu)\mathbf{i} + (2\mu^2-5\mu)\boldsymbol{k}$ $\left(\overrightarrow{OA}\times\overrightarrow{OB}\right)\cdot\overrightarrow{OC} = \left((5-2\mu)\mathbf{i}+(2\mu^2-5\mu)\boldsymbol{k}\right)\cdot(4\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ $= 20-8\mu+6\mu^2-15\mu$ $= 6\mu^2-23\mu+20$ $= (2\mu-5)(3\mu-4)$ O , A, B , C are coplanar if and only if $\left(\overrightarrow{OA}\times\overrightarrow{OB}\right)\cdot\overrightarrow{OC} = 0$ $\mu = \frac{5}{2}$ or $\mu = \frac{4}{3}$		

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4	(a)	$\lvert x-y\rvert < \lvert 1-\bar{x}y\rvert$ $\Leftrightarrow \lvert x-y\rvert^2 < \lvert 1-\bar{x}y\rvert^2$ $\Leftrightarrow (x-y)\overline{(x-y)} < (1-\bar{x}y)\,\overline{(1-\bar{x}y)}$ $\Leftrightarrow (x-y)(\bar{x}-\bar{y}) < (1-\bar{x}y)\,(\bar{1}-\overline{\bar{x}y})$ $\Leftrightarrow (x-y)(\bar{x}-\bar{y}) < (1-\bar{x}y)\,(1-\bar{\bar{x}}\bar{y})$ $\Leftrightarrow (x-y)(\bar{x}-\bar{y}) < (1-\bar{x}y)\,(1-x\bar{y})$ $\Leftrightarrow \lvert x\rvert^2 - x\bar{y} - \bar{x}y + \lvert y\rvert^2 < 1 - \bar{x}y - x\bar{y} + \lvert x\rvert^2\lvert y\rvert^2$ $\Leftrightarrow 0 < 1 - \lvert x\rvert^2 - \lvert y\rvert^2 + \lvert x\rvert^2\lvert y\rvert^2$ $\Leftrightarrow 0 < (1-\lvert x\rvert^2) - \lvert y\rvert^2(1-\lvert x\rvert^2)$ $\Leftrightarrow 0 < (1-\lvert x\rvert^2)(1-\lvert y\rvert^2)$ $\Leftrightarrow \{0 < (1-\lvert x\rvert^2)$ and $0 < (1-\lvert y\rvert^2)\}$ or $\{0 > (1-\lvert x\rvert^2)$ and $0 > (1-\lvert y\rvert^2)\}$ $\Leftrightarrow \{1 < \lvert x\rvert^2$and $1 < \lvert y\rvert^2\}$ or $\{1 > \lvert x\rvert^2$and $1 > \lvert y\rvert^2\}$ $\Leftrightarrow \{1 < \lvert x\rvert$and $1 < \lvert y\rvert\}$ or $\{1 > \lvert x\rvert$ and $1 > \lvert y\rvert\}$	75	

QNo		Answer	Marks	Comments
4	(b)	De Moivre's theorem $\forall\, n \in Q \;\; (cos\theta + isin\theta)^n = cosn\theta + isinn\theta$ $z = cos\theta + isin\theta$ $z^n = cosn\theta + isinn\theta$ $\frac{1}{z^n} = \frac{1}{cosn\theta + isinn\theta} = cosn\theta - isinn\theta$ $z^n + \frac{1}{z^n} = 2cosn\theta \quad z^n - \frac{1}{z^n} = 2isin\theta$	40	
	(c)	When $n = 1$ $z + \frac{1}{z} = 2cos\theta \quad z - \frac{1}{z} = 2isin\theta$ $2^4cos^4\theta = \left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4}$ $16cos^4\theta = z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $16cos^4\theta = 2cos4\theta + 4.2cos2\theta + 6$ $8cos^4\theta = cos4\theta + 4cos2\theta + 3$ $2^4i^4sin^4\theta = \left(z - \frac{1}{z}\right)^4$ $= z^4 + 4z^3\left(-\frac{1}{z}\right) + 6z^2\left(-\frac{1}{z}\right)^2 + 4z\left(-\frac{1}{z}\right)^3$ $+\left(-\frac{1}{z}\right)^4$ $16sin^4\theta = z^4 + \frac{1}{z^4} - 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $16sin^4\theta = 2cos4\theta - 4.2cos2\theta + 6$ $8cos^4\theta = cos4\theta - 4cos2\theta + 3$	45	
	(c)	$Re(z + a) = \|z - a\|$ Let $z = x + iy$ where $x, y \in R$ $Re(x + iy + a) = \|x + iy - a\|$ $(x + a) = \sqrt{(x - a)^2 + y^2}$ if $(x + a) \geq 0$ Taking the square of both sides $y^2 = 4ax$ Then it is parabola	40	

Q No		Answer	Marks	Comments
5	(a)	$\frac{dy}{dx}=\frac{x+3y}{3x+y}$ Substitute $y=vx$ $\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$ $v+x\frac{dv}{dx}=\frac{x+3vx}{3x+vx}\quad x\frac{dv}{dx}=\frac{1+3v}{3+v}-v\quad x\frac{dv}{dx}=\frac{1+3v-3v-v^2}{3+v}$ $\frac{3+v}{1-v^2}dv=\frac{1}{x}dx$ $\frac{3+v}{1-v^2}=\frac{A}{1+v}+\frac{B}{1-v}\quad 3+v=A(1-v)+B(1+v)$ Taking $v=1$ and $v=-1$ we get $B=2$ and $A=1$ respectively $\left(\frac{1}{1+v}+\frac{2}{1-v}\right)dv=\frac{1}{x}dx$ $\ln\lvert 1+v\rvert-2\ln\lvert 1-v\rvert=\ln\lvert x\rvert+\ln\lvert c\rvert$ where c is an arbitrary constant $\ln\left\lvert\frac{1+v}{(1-v)^2}\right\rvert=\ln\lvert cx\rvert\quad \left\lvert\frac{1+v}{(1-v)^2}\right\rvert=\lvert cx\rvert$ $\frac{1+v}{(1-v)^2}=\pm cx$ take $\lambda=\pm c$ $\quad \frac{1+\frac{y}{x}}{\left(1-\frac{y}{x}\right)^2}=\lambda x$ $(x+y)=\lambda(x-y)^2$	80	
	(b)	$2x(2x+3y^2)\frac{dy}{dx}+y(3x+2y^2)=0$ $(4x^2+6xy^2)\frac{dy}{dx}+(3xy+2y^3)=0$ Take $f(x,y)=3xy+2y^3$ and $g(x,y)=4x^2+6xy^2$ $\frac{\partial f}{\partial y}=3x+6y^2\quad \frac{\partial g}{\partial x}=8x+6y^2\quad \frac{\partial f}{\partial y}\neq\frac{\partial g}{\partial x}$ The equation is not exact	20	

Q No		Answer	Marks	Comments
		Multiplying by $x^h y^k$ $(4x^{h+2}y^k + 6x^{h+1}y^{k+2})\frac{dy}{dx} + (3x^{h+1}y^{k+1} + 2x^h y^{k+3}) = 0$ Take $Q(x,y) = (4x^{h+2}y^k + 6x^{h+1}y^{k+2} \quad P(x,y) = 3x^{h+1}y^{k+1} + 2x^h y^{k+3}$ $\frac{\partial P}{\partial y} = 3(k+1)x^{h+1}y^k + 2(k+3)x^h y^{k+2}$ $\frac{\partial Q}{\partial x} = 4(h+2)x^{h+1}y^k + 6(h+1)x^h y^{k+2}$ If the equation is exact then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ Equating the coefficients of $x^{h+1}y^k \ and \ x^h y^{k+2}$ $3k+3 = 4h+8 \quad 6h+6 = 2k+6 \ \ h=1 \ and \ k=3$ xy^3 is integrating factor $(4x^3y^3 + 6x^2y^5)\frac{dy}{dx} + (3x^2y^4 + 2xy^6) = 0$ Is exact. If $U(x,y)$ is a solution $\frac{\partial U}{\partial x} = 3x^2y^4 + 2xy^6$, $\frac{\partial U}{\partial y} = 4x^3y^3 + 6x^2y^5$ Consider $\frac{\partial U}{\partial x} = 3x^2y^4 + 2xy^6$ Integrating w.r.t. x $U(x,y) = x^3y^4 + x^2y^6 + f(y)$ Where $f(y)$ is arbitrary function of y. $\frac{\partial U}{\partial y} = 4x^3y^3 + 6x^2y^5 + \frac{d(f(y))}{dy}$ $\frac{d(f(y))}{dy} = 0$ then $f(y) = cons\tan t \ c$ The fore the solution is $U(x,y) = x^3y^4 + x^2y^6 + c$	100	

Q No		Answer	Marks	Comments
6	(a)	Take $f(x) = x^3 - 2$ $f(1) = -1$, $f(2) = 6$ and is continues in $[1, 2]$ there exists a root in the interval $[1, 2]$.	15	
		$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$ $x = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$ $3x = 2x + \frac{2}{x^2}$ $x = \frac{2}{x^2}$ $x^3 = 2$ $x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$ is a simple iteration formula to evaluate a numerical value for $\sqrt[3]{2}$ Consider $g(x) = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$ $g'(x) = \frac{2}{3}\left(1 - \frac{2}{x^3}\right)$ Since $1 \le x \le 2$ $1 \le x^3 \le 8$ $2 \ge \frac{1}{x^3} \ge \frac{1}{4}$ $-1 \le 1 - \frac{1}{x^3} \le \frac{3}{4}$ $-\frac{2}{3} \le \frac{2}{3}\left(1 - \frac{1}{x^3}\right) \le \frac{1}{2}$ $-\frac{2}{3} \le f'(x) \le \frac{1}{2}$ Clearly $\lvert f'(x) \rvert < 1$ x_n converges to $\sqrt[3]{2}$	40	
		$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$ $x_2 = \frac{2}{3}\left(x_1 + \frac{1}{x_1^2}\right) = \frac{2}{3}\left(1 + \frac{1}{1^2}\right) = \frac{4}{3} = 1 \cdot 333333$ $x_3 = \frac{2}{3}\left(x_2 + \frac{1}{x_2^2}\right) = \frac{2}{3}\left(1 \cdot 33333 + \frac{1}{1 \cdot 33333^2}\right) = 1 \cdot 26389$	30	

Q No		Answer	Marks	Comments
		$x_4 = \frac{2}{3}\left(x_3 + \frac{1}{x_3^2}\right) = \frac{2}{3}\left(1\cdot 26389 + \frac{1}{1\cdot 26389^2}\right) = 1\cdot 25993$ $x_5 = \frac{2}{3}\left(x_4 + \frac{1}{x_4^2}\right) = \frac{2}{3}\left(1\cdot 25993 + \frac{1}{1\cdot 25993^2}\right) = 1\cdot 25992$ $x_6 = \frac{2}{3}\left(x_5 + \frac{1}{x_5^2}\right) = \frac{2}{3}\left(1\cdot 25992 + \frac{1}{1\cdot 25992^2}\right) = 1\cdot 25992$ Therefore the numerical to correct to two decimal places of $\sqrt[3]{2}$ is $1\cdot 26$		
	(b)	$10x + y - 2z = 7$ $3x + 10y - z = -8$ $2x - 3y + 10z = 15$ $x = \frac{1}{10}(7 - y + 2z)$ $y = \frac{1}{10}(-8 - 3x + z)$ $z = \frac{1}{10}(-2x + 3y + 15)$ $x_{n+1} = \frac{1}{10}(7 - y_n + 2z_n)$ $y_{n+1} = \frac{1}{10}(-8 - 3x_n + z_n)$ $z_{n+1} = \frac{1}{10}(15 - 2x_n + 3y_n)$ $x_0 = y_0 = z_0 = 0$ $x_1 = 0\cdot 7$, $y_1 = -0\cdot 8$, $z_1 = 1\cdot 5$ $x_2 = \frac{1}{10}(7 + 0\cdot 8 + 3) = 1\cdot 08$ $y_2 = \frac{1}{10}(-8 - 2\cdot 1 + 1\cdot 5) = -0\cdot 86$ $z_2 = \frac{1}{10}(15 - 1\cdot 4 + 2\cdot 4) = 1\cdot 12$ $x_3 = \frac{1}{10}(7 + 0\cdot 86 + 2\cdot 24) = 1\cdot 01$ $y_3 = \frac{1}{10}(-8 - 3\cdot 24 + 1\cdot 12) = -1\cdot 012$ $z_3 = \frac{1}{10}(15 - 2\cdot 16 - 2\cdot 58) = 1\cdot 026$	115	

Q No		Answer	Marks	Comments
		$x_4 = \frac{1}{10}(7 + 1\cdot012 + 2\cdot052) = 1\cdot0064$ $y_4 = \frac{1}{10}(-8 - 3\cdot03 + 1\cdot026) = -1\cdot0004$ $z_4 = \frac{1}{10}(15 - 2\cdot02 - 3\cdot036) = 0\cdot9944$ $x_5 = \frac{1}{10}(7 + 1\cdot0004 + 1\cdot9888) = 0\cdot99892$ $y_5 = \frac{1}{10}(-8 - 3\cdot0192 + 0\cdot9944) = -1\cdot00248$ $z_5 = \frac{1}{10}(15 - 2\cdot0128 - 3\cdot0012) = 0\cdot9986$ $x_6 = \frac{1}{10}(7 + 1\cdot00248 + 1\cdot9972) = 0\cdot999968$ $y_5 = \frac{1}{10}(-8 - 2\cdot99676 + 0\cdot9986) = -0\cdot999816$ $z_5 = \frac{1}{10}(15 - 1\cdot99784 - 3\cdot00744) = 0\cdot999472$		

Q No		Answer	Marks	Comments
7	(a)	Definition of mean and variance $\int_{-\propto}^{\propto} f(x)dx = 1$ $\int_0^3 \lambda x(3-x)(3+x)dx = 1$ $\lambda \int_0^3 (9x - x^3)dx = 1$ $\lambda \left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3 = 1$ $\lambda \left[\frac{81}{2} - \frac{81}{4}\right] = 1$ $\lambda = \frac{4}{81}$	35	

Q No		Answer	Marks	Comments
		Theory part	10	
		$E(X) = \int_{-\propto}^{\propto} f(x)dx$ $= \int_{0}^{3} x \frac{4}{81}(9x - x^3)dx$ $= \frac{4}{81}\left[\frac{9x^3}{3} - \frac{x^5}{5}\right]_0^3$ $= \frac{4}{81}\left[\frac{243}{3} - \frac{243}{5}\right]$ $= \frac{8}{5} = 1 \cdot 6$ $E(x^2) = \int_{-\propto}^{\propto} x^2 f(x)dx$ $= \int_{0}^{3} x^2 \frac{4}{81}(9x - x^3)dx$ $= \frac{4}{81}\left[\frac{9x^4}{4} - \frac{x^6}{6}\right]_0^3 = \frac{4}{81}\left[\frac{9 \cdot 81}{3} - \frac{81 \cdot 9}{5}\right] = 3$ $V(X) = E(X^2) - (E(X))^2$ $V(X) = 3 - \left(\frac{8}{5}\right)^2 = 0 \cdot 44$	45	
		$f(x) = \lambda(9x - x^3)$ $f^1(x) = \lambda(9 - 3x^2)$ $f'^1(x) = -3\lambda(x - \sqrt{3})(x + \sqrt{3})$ Since $x \in [0,3]$ $f'^1(x) = 0 \leftrightarrow x = \sqrt{3}$ $f^2(x) = -6\lambda x$ $f^2(\sqrt{3}) = -6\lambda\sqrt{3} < 0$ $\therefore$ At $x = \sqrt{3}$ $f(x)$ gets the maximum value $\therefore$ The mode is $\sqrt{3}$.	30	First order derivative method can be used.

<table>
<tr><th colspan="2">Q No</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td></td><td></td><td>Suppose the median is m then

$$\int_{-\propto}^{m} f(x)dx = \frac{1}{2}$$

$$\int_{0}^{m} \frac{4}{81}(9x - x^3)dx = \frac{1}{2}$$

$$\frac{4}{81}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^m = \frac{1}{2}$$

$$\frac{4}{81}\left[\frac{9m^2}{2} - \frac{m^4}{4}\right] = \frac{1}{2}$$

$$m^4 - 36m^2 + 81 = 0$$

$$m^2 = \frac{-36 \pm \sqrt{36^2 - 4.2.81}}{2.2}$$

$$= 9 \pm \frac{9\sqrt{2}}{2}$$

Since $0 \leq m^2 \leq 9$ $\quad m^2 = 9 - \frac{9\sqrt{2}}{2} \quad m = 1 \cdot 62$</td><td>45</td><td></td></tr>
<tr><td></td><td>(b)</td><td>To State the Bayes theorem.</td><td>15</td><td></td></tr>
<tr><td></td><td></td><td>A_r = The event that randomly selected urn is U_r.

$$P(A_r) = \frac{1}{13} \quad r = 1, 2, 3, \dots \dots .13$$

A = The event that randomly chosen ball is red.

$P\left({}^{A}/_{A_r}\right) = \frac{r}{13}$ where $r = 1, 2, 3, \dots \dots .13$

From the Bay's theorem

$$P\left({}^{A_9}/_{A}\right) = \frac{P(A_9)P\left({}^{A}/_{A_9}\right)}{\sum_{r=1}^{13} P(A_9)P\left({}^{A}/_{A_r}\right)}$$

$$P\left({}^{A_9}/_{A}\right) = \frac{\frac{1}{13} \cdot \frac{9}{13}}{\sum_{r=1}^{13} \frac{1}{13} \cdot \frac{r}{13}} = \frac{9}{\sum_{r=1}^{13} r} = \frac{9}{\frac{13(13+1)}{2}} = \frac{9}{91}$$

$$= 0 \cdot 0989$$</td><td>30</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">Q No</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td rowspan="2">8</td><td rowspan="2">(a)</td><td>

$$y = \frac{x^2 + x + 2}{x - 1}$$

Clearly $x = 1$ is a vertical asymptote.

When $x \neq 0$

$$m = \frac{y}{x} = \frac{x+1+\frac{2}{x}}{x-1} = \frac{1+\frac{1}{x}+\frac{2}{x^2}}{1-\frac{1}{x}} \quad m = \lim_{x\to\pm\propto}\left(\frac{y}{x}\right) = 1$$

$$\therefore m = 1$$

$$\lim_{x\to\pm\propto}(y - mx) = \lim_{x\to\pm\propto}\left(\frac{x^2+x+2}{x-1} - x\right)$$

$$= \lim_{x\to\pm\propto}\left(\frac{2x+2}{x-1}\right) = \lim_{x\to\pm\propto}\left(\frac{2+\frac{2}{x}}{1-\frac{1}{x}}\right) = 2$$

$\lim_{x\to\infty}(y - mx) = 2 \therefore c = 2 \quad \therefore y = x + 2$ is an asymptote.

$$\frac{dy}{dx} = \frac{(x-1)(2x-1) - (x^2 + x + 2)}{(x-1)^2}$$

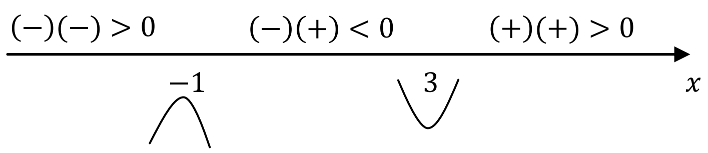
$$\frac{dy}{dx} = \frac{2x^2 - x - 1 - x^2 - x - 2}{(x-1)^2} = \frac{x^2 - 2x - 3}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{(x-3)(x+1)}{(x-1)^2}$$

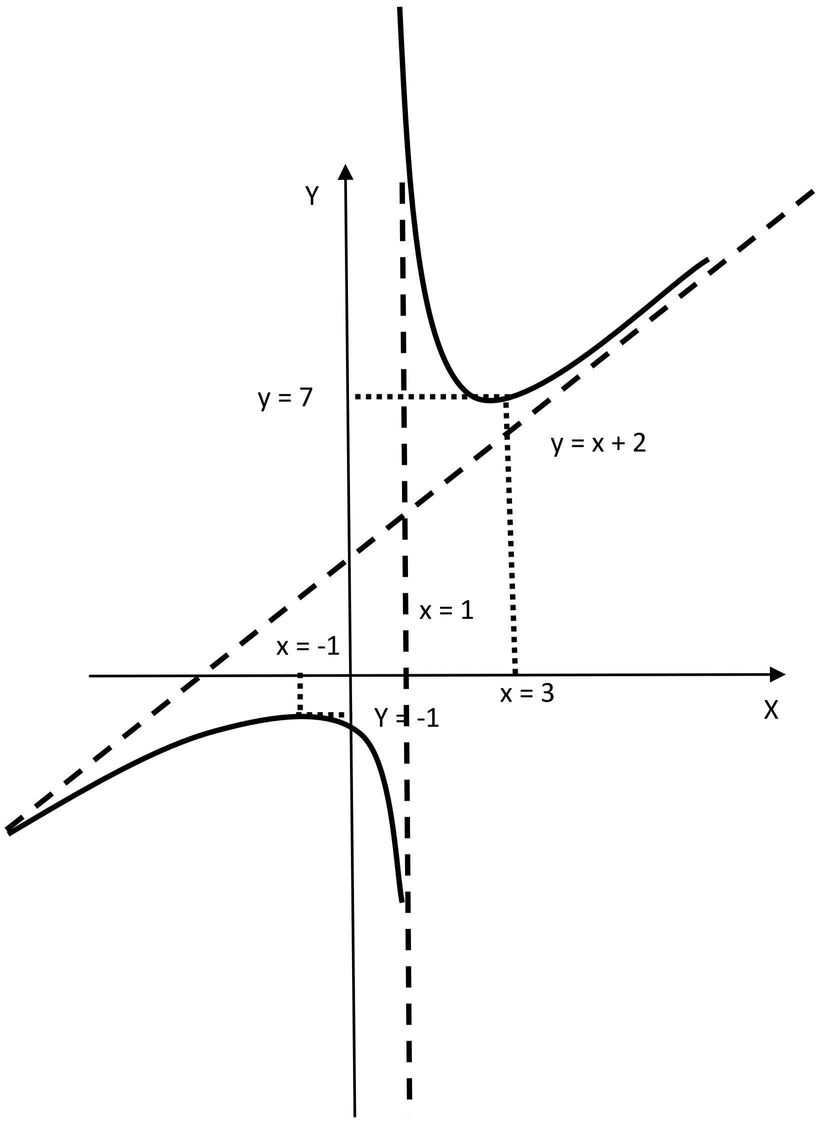
$$\frac{dy}{dx} = 0 \Leftrightarrow x = -1 \text{ or } x = 3$$

</td><td rowspan="2">90</td><td></td></tr>
<tr><td>

The sign of $\frac{dy}{dx}$



</td><td>Second order derivative method can be used to find maxima and minima.</td></tr>
</table>

Q No		Answer	Marks	Comments
		When x = -1, y = -1 when y = 3, y = 7 (-1,-1) (3, 7) The sign of $\frac{dy}{dx}$ When x = 0 y = -2 (0, -2) When $x \neq 0$ $y = \frac{x\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}{\left(1-\frac{1}{x}\right)}$ $\therefore \lim_{x\to\infty} y = \infty\ and\ \lim_{x\to-\infty} y = -\infty$ $y = \frac{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}{(x-1)} \therefore\ y \neq 0$  		

Q No		Answer	Marks	Comments
	(b)	(i) $z = ln(x^2 + y^2)$ $$\frac{\partial z}{\partial x} = \frac{1}{x^2 + y^2} 2x$$ $$\frac{\partial^2 z}{\partial x^2} = 2\frac{(x^2 + y^2).1 - x(2x)}{x^2 + y^2}$$ $\frac{\partial^2 z}{\partial x^2} = 2\frac{(y^2 - x^2)}{(x^2+y^2)^2}$ …………..[1] $$z = ln(x^2 + y^2)$$ $$\frac{\partial z}{\partial y} = \frac{1}{x^2 + y^2} 2y$$ $$\frac{\partial^2 z}{\partial y^2} = 2\frac{(x^2 + y^2).1 - y(2y)}{(x^2 + y^2)^2}$$ $\frac{\partial^2 z}{\partial y^2} = 2\frac{(x^2 - y^2)}{(x^2+y^2)^2}$ …………..[2] Clearly $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 y}{\partial x^2} = 0$	55	
		(ii) $u = ln(x^2 + xy + y^2)$ $y = e^x$ $\frac{dy}{dx} = e^x$ Since u is a function of x and y $du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$ $$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$$ $$\frac{\partial u}{\partial x} = \frac{1}{x^2 + xy + y^2}(2x + y)$$ $$\frac{\partial u}{\partial x} = \frac{2x + y}{x^2 + xy + y^2}$$ $$\frac{\partial u}{\partial y} = \frac{1}{x^2 + xy + y^2}(2y + x)$$ $$\frac{\partial u}{\partial y} = \frac{2y + x}{x^2 + xy + y^2}$$ $$\frac{du}{dx} = \frac{2x + y}{x^2 + xy + y^2} + \frac{2y + x}{x^2 + xy + y^2} e^x$$ $$\frac{du}{dx} = \frac{(2x + y) + (2y + x)e^x}{x^2 + xy + y^2}$$	55	